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$$\begin{array}{lll} x_1 = +0.031 & \qquad & y_1 = -0.285 \\ x_2 = +0.028 & & y_2 = 0.090 \\ x_3 = +0.031 & & y_3 = -0.355 \end{array}$$

Substitute these values in the observation equations and the residuals result. Squaring and adding,

$$[vv] = 0.0003.$$

Hence for the p. e. of a single observation

$$r = .6745\sqrt{\tfrac{.0003}{3}} = \pm 0.0007.$$

$$\begin{array}{rl} \therefore \text{ p. e. of } x_1 & = .007\sqrt{.35} = 0^\circ.004 \\ \text{“ \quad “ } x_2 & = .007\sqrt{.3} = 0^\circ.004 \\ \text{“ \quad “ } x_3 & = .007\sqrt{.35} = 0^\circ.004 \end{array}$$

Second Solution. By Neumann's formula—Arranging according to table

z_1	z_2	z_3	z_4	Sums
	—.03	—.02	—.07	—.12
		—.01	—.02	—.03
			—.03	—.03
+.12	+.03	+.03		
+.12	00	00	—.12	

$$\begin{array}{lll} \therefore z_1 = +0^\circ.03 & \qquad & z_3 = +0^\circ.00 \\ z_2 = +0^\circ.00 & & z_4 = -0^\circ.03 \end{array}$$

$$\begin{array}{rcl} \therefore x_1 = z_1 & & = 0^\circ.03 \\ x_2 = x_1 + z_2 & & = 0\ .03 \\ x_3 = x_2 + z_3 & & = 0\ .03 \end{array}$$

Also

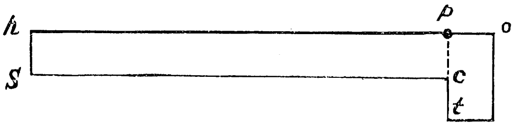
$$r = .6745\sqrt{\tfrac{.0004}{3}} = \pm 0.007$$

$$\begin{array}{rl} \text{and p. e. of } x_1 & = .007\sqrt{\tfrac{1}{3}} = \pm 0^\circ.004 \\ \text{“ \quad “ } x_2 & = .007\sqrt{\tfrac{1}{4}} = \pm 0\ .004 \\ \text{“ \quad “ } x_3 & = .007\sqrt{\tfrac{1}{3}} = \pm 0\ .004. \end{array}$$

THE MULTISECTION OF ANGLES.

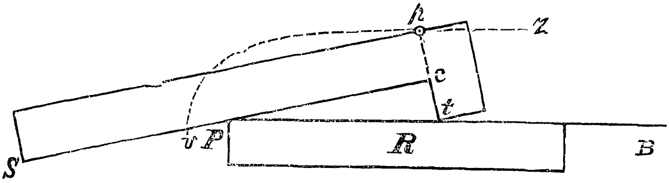
BY PROF. J. W. NICHOLSON, A. M., LA. STATE UNIV., BATON ROUGE, LA.

To divide an angle into any number of equal parts, we use an instrument called the *multisector*, the simplest form of which is the square *sot;* which consists of two perpendicular arms *os* and *ct*, the length (*ct*) of the latter being equal to the width (*cp*)



of the former. At p, the intersection of ro and the prolongation of tc, is an attachment for carrying a pencil.

THE POLYODE.—If we place the multisector on the rule R, or on the line AB, as in the diag'm, and slide it along on the rule or line so that sc will continua'y pass through the point P, while t continues along the line BP, the pencil at p will describe the curve zpv, which is called a *polyode*, from πολυσ, many, and οδοσ, path.



The line PB, along which the multisector moves, is the *directrix*, and may be either straight or curved.

The point P, through which the side sc continually passes is the *pole*, and may be either on or without the directrix.

The line, or distance, ct or cp, is the *modulus* of the multisector.

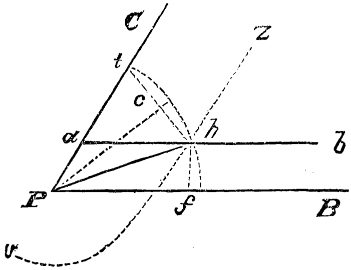
Let us now apply the multisector, or poliode, to the dividing of an angle into any number of equal parts.—1st, *To trisect an angle.*

Let CPB be the given angle. Draw ab parallel to PB and at a distance from it equal to the modulus of the multisector. With PC as a directrix, and P as a pole, describe the polyode vz cutting ab at p. Draw Pp, and we shall have

$$\angle BPp = \tfrac{1}{3}\angle BPC.$$

For, evidently

$$\angle tPp = 2\angle pPf.$$

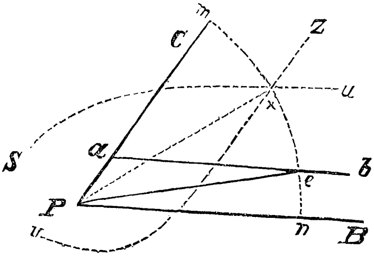


2nd. *To divide an angle into five equal parts.*

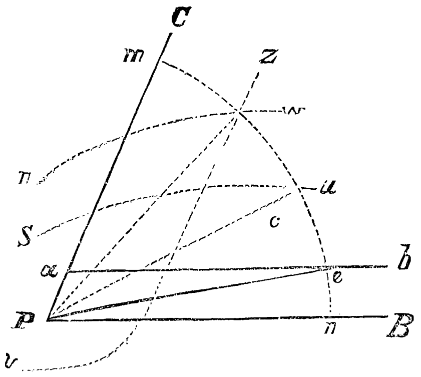
Let CPB be the given angle. Draw ab and vz as in the preceding solution. With ab as a directrix and P as a pole, describe the polyode su cutting vz in x. From P as a center, with a radius $= Px$, describe the arc mn cutting ab at e. Draw Pe, and we shall have

$$\angle BPe = \tfrac{1}{5}\angle BPC.$$

For, evidently $\quad$ ang. mPx = ang. xPe = 2ang. ePn.



3rd. *To divide an angle into seven equal parts.*



Let CPB be the given angle. Draw ab, vz and su as in the preceding solution. With su as a directrix, and P as a pole, describe the polyode rw cutting vz in x. From P as a center, with a radius $= Px$, describe the arc mn cutting ab at e, and su at c. Draw Pe and we sh'l evidently have

$$\text{ang.}\, BPe = \tfrac{1}{7}\, \text{ang.}\, BPC.$$

In a similar manner we may const.

$$\text{ang.}\, BPe = \tfrac{1}{2n+1}\, \text{ang.}\, BPC.$$

EQUATION OF THE POLYODE.—The equation of the polyode evidently depends on the nature of thr directrix and the position of the pole; and hence may have an indefinite number of forms. Denoting the coordinates of the pole by a and b, the modulus by m, and the equation of the directrix by

$$y_1 = f(x_1), \qquad (1)$$

the eq. of the polyode may be found by combin'g (1) with the foll. relat'ns;

$$(y-b)^2+(y-a)^2 = (y_1-b)^2+(x_1-a)^2 \;..(2),\quad (y-y_1)^2+(x_1-x)^2 = 4m^2. \;(3)$$

Combining (2) and (3), we find, say,

$$y_1 = \varphi(x, y), \qquad x_1 = \theta(x, y).$$

Substituting, $\quad \varphi(x, y) = f[\theta(x, y)]. \qquad (4).$

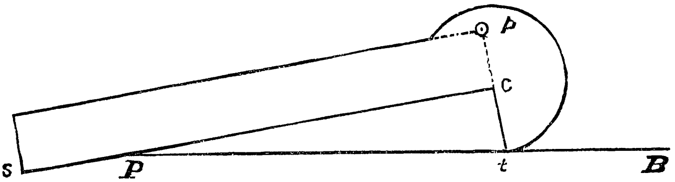
If the directrix coincides with the axis of X, or (1) is of the form $y_1 = 0x_1$, and $a = b = 0$, (4) becomes $x^2+y^2--x(x^2+y^2)^{\frac{1}{2}}=2m^2$, or $x(4m^2--y^2)^{\frac{1}{2}} = 2m^2--y^2$; which is the equation of vz in the 2nd Fig. above, regarding PB as the axis of X, and P the origin. This is the *simplest* form of the polyode, and is identically the same curve as that so ingeniously empl'd by Dr. Hillhouse in trisectig an angle (ANALYST, Vol. IX, No. 6).

Again if (1) is of the form $y_1 = 0x_1 + m$, and $a = b = 0$, (4) becomes

$$(y-m)^2+[\sqrt{(y^2+x^2-m^2)}-x]^2 = 4m^2;$$

which is the eq'n of su in the 4th Fig., PB being axis of X, and origin at P

A multis'r of this model may be made of pasteboard and used to advantage in the class-r'm.



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